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Novel Micromechanics-Based Woven-Fabric Composite Constitutive Model with Material Nonlinear Behavior

Ala Tabiei,* Yiwei Jiang,† and Witao Yi‡
University of Cincinnati, Ohio 45221-0070

An averaging procedure is presented for analysis of plain-weave-fabric composites with material nonlinear behavior. A representative volume cell is assumed. The cell is divided into many subcells, and averaging is performed again. The effective stress-strain relations of the subcell are obtained. Using iso-stress and iso-strain assumptions, the constitutive equations are averaged along the thickness direction. As a result, a linear system of equations is formed and subsequently solved to distribute the global average strains to each subcell. In this manner, given the average strains, the global average stresses can be determined. Numerical results are generated for plain-weave composite with material nonlinear behavior.

Nomenclature

a = half-length of the representative cell
b = half-width of the representative cell
E = Young's modulus
G = shear modulus
H = height of the representative cell
H_t = height of fiber tows
S_{ij} = compliance components in principal material system
$\underset{\sim}{S}_{ij}$ = compliance components of constituents in global coordinate system
θ_f = local angle between the fill yarn and global coordinate system
θ_w = local angle between the warp yarn and global coordinate system
$(\bar{\ })$ = average quantities of representative volume cell

Subscripts

f = quantities of the fill tow
i = in-plane stress or strain components
m = quantities of the matrix
o = out-of-plane stress or strain components
w = quantities of the warp tow
x, y, z = quantities in global coordinate system
1, 2, 3 = quantities in principal material coordinates

Superscripts

t = quantities of tangential stiffness matrix or compliance matrix
(α, β) = average quantities of a subcell
$*$ = quantities in principal material coordinate system

Introduction

WOVEN fabric composites have long been recognized as more competitive than unidirectional composites for their good stability in the mutually orthogonal warp and fill directions. This is attributed to more balanced properties in the fabric plane and enhanced impact resistance. These advantages have resulted in a growing interest in the use of woven fabric composites for structural applications and the development of analytical procedures for prediction of the thermomechanical properties of these woven fabric composites. Analytical models for determination of mechanical properties of woven composites provide a cost-effective tool to determine the effects of several parameters on the mechanical properties of these composites. Some of these parameters include fabric weight, constituent volume fraction, yarn undulation, weave style and properties of the constituent materials.

The finite element method can be used to study the overall behavior of composite structures on the macro level and the material behavior on the constituent level. Whitcomb,[1] Zhang and Harding,[2] Chapman and Whitcomb,[3] and many other researchers have studied the elastic material properties of woven composites by the finite element method. However, the concentration was on obtaining the elastic properties of woven composites. A more practical issue can arise when studying the global behavior of structures with consideration of the material nonlinearity. Consequently, application of a micromechanics-based material model into finite element analysis solvers provides a feasible way in dealing with the global behavior of woven composite structures.

A number of analytical models were presented by many investigators. All of these models studied the average performance of a periodic representative volume cell. Ishikawa and Chou suggested a variety of models[4–6] to handle the in-plane behavior of woven composites. These are the mosaic model, the fiber undulation model, and the bridging model. The basic assumption for these models is that the classical lamination theory is valid for every infinitesimal strip of a representative cell. Ishikawa and Chou[7] also expanded their models to deal with constituent materials with shear nonlinearity and initial failure (known as knee phenomenon). Their work basically considers a one-dimensional strip of a representative cell. As a result, these models cannot represent the material behavior of woven composites under bidirectional loading. More recently, Naik and Colleagues[8–11] proposed a series-parallel model and a parallel-series model for considering the two-dimensional undulation geometry of plain woven composites with linear and nonlinear material behavior. More recently Nayfeh et al.[12] developed a model based on the micromechanical behavior of a representative unit cell. The global constitutive relationships were consistently derived from the total strain energy of the system.

The developed analytical models to date provide a reasonable prediction of material property. However, there is a need to develop an incremental stress-strain procedure for woven composites with material nonlinear effects. In this research a homogenization procedure is presented that provides the constitutive equations of plain woven fabric composites, with material nonlinearity, from the constitutive relations at the constituent level. It is practically important to use the proposed method when global analysis of structures can directly require the constitutive equations of the constituents. Global analysis such as strength modeling, temperature effect, and strain-rate effect would benefit greatly from such formulation.

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*Assistant Professor, Aerospace Engineering and Engineering Mechanics Department.
†Graduate Research Assistant, Aerospace Engineering and Engineering Mechanics Department; currently Product Engineer, Hamilton Sundstrand.
‡Graduate Research Assistant, Aerospace Engineering and Engineering Mechanics Department.

Mathematical Formulation

Any nonhybrid plain-weave-fabric composite lamina can be represented by double-periodic representative volume cells, as shown in Fig. 1. The representative volume cell typically consists of two sets of interlaced yarns, known as fill and warp threads, and resin. The fill and warp threads, in the form of mixture of fibers and resin, are assumed to be homogenous and transversely isotropic. The resin is homogenous and isotropic. Because of symmetry, the representative volume cell is divided into a quarter cell. The quarter cell is further divided into infinitesimal blocks as shown in Fig. 1. Consider a typical infinitesimal block of the cell with dx in length, dy in width, and H in height. This infinitesimal element generally consists of three different materials with $\bar{t}_m$, $\bar{t}_f$, and $\bar{t}_w$ volume fractions of matrix, fill, and warp, respectively. The material coordinate system of fills and warps may not coincide with the global coordinate system. The effective incremental average stress and strain vector can be expressed by the following equations:

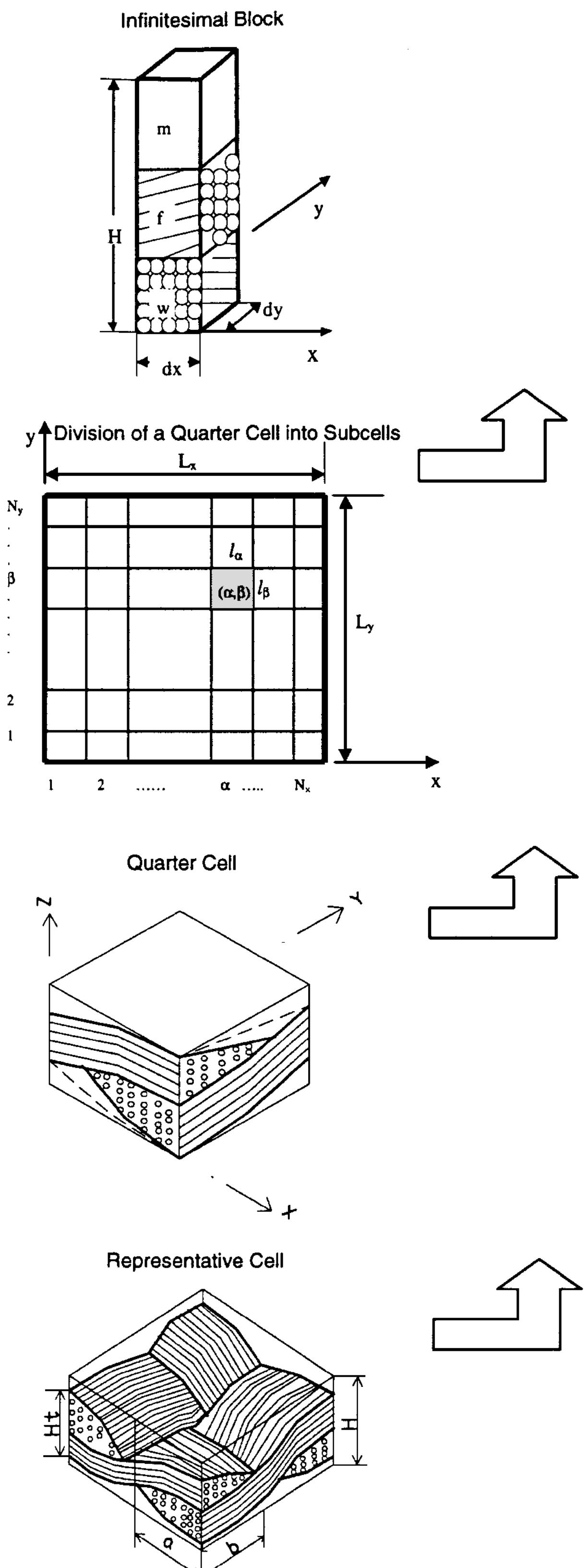

Fig. 1 Representative volume cell and corresponding divisions.

$$\mathrm{d}\{\varepsilon_i\} = \mathrm{d}\{\varepsilon_i\}_k \tag{1}$$

$$\mathrm{d}\{\varepsilon_o\} = \sum_{k=m,f,w} \bar{t}_k \,\mathrm{d}\{\varepsilon_o\}_k \tag{2}$$

$$\mathrm{d}\{\sigma_i\} = \sum_{k=m,f,w} \bar{t}_k \,\mathrm{d}\{\sigma_i\}_k \tag{3}$$

$$\mathrm{d}\{\sigma_o\} = \mathrm{d}\{\sigma_o\}_k \tag{4}$$

where $\{\varepsilon_i\}$, $\{\varepsilon_o\}$, $\{\sigma_i\}$, and $\{\sigma_o\}$ are the in-plane strains, out-of-plane strains, in-plane stresses, and out-of-plane stresses, respectively. The compliance matrix is given by the following equation:

$$\left\{\begin{array}{c} \mathrm{d}\varepsilon_i^* \\ \hline \mathrm{d}\varepsilon_o^* \end{array}\right\}_k = \left[\begin{array}{c|c} S_i^t & S_{io}^t \\ \hline S_{oi}^t & S_o^t \end{array}\right]_k \left\{\begin{array}{c} \mathrm{d}\sigma_i^* \\ \hline \mathrm{d}\sigma_o^* \end{array}\right\}_k \tag{5}$$

where

$$\mathrm{d}\{\varepsilon_i^*\} = \begin{Bmatrix} \mathrm{d}\varepsilon_{11} \\ \mathrm{d}\varepsilon_{22} \\ \mathrm{d}\gamma_{12} \end{Bmatrix}, \qquad \mathrm{d}\{\varepsilon_o^*\} = \begin{Bmatrix} \mathrm{d}\varepsilon_{33} \\ \mathrm{d}\gamma_{13} \\ \mathrm{d}\gamma_{23} \end{Bmatrix}$$

$$\mathrm{d}\{\sigma_i^*\} = \begin{Bmatrix} \mathrm{d}\sigma_{11} \\ \mathrm{d}\sigma_{22} \\ \mathrm{d}\sigma_{12} \end{Bmatrix}, \qquad \mathrm{d}\{\sigma_o^*\} = \begin{Bmatrix} \mathrm{d}\sigma_{33} \\ \mathrm{d}\sigma_{13} \\ \mathrm{d}\sigma_{23} \end{Bmatrix}$$

$$[S_i^t] = \begin{bmatrix} S_{11}^t & S_{12}^t & 0 \\ S_{12}^t & S_{22}^t & 0 \\ 0 & 0 & S_{66}^t \end{bmatrix}, \qquad [S_{io}^t] = \begin{bmatrix} S_{13}^t & 0 & 0 \\ S_{23}^t & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[S_{oi}^t] = \begin{bmatrix} S_{13}^t & S_{23}^t & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad [S_o^t] = \begin{bmatrix} S_{33}^t & 0 & 0 \\ 0 & S_{55}^t & 0 \\ 0 & 0 & S_{44}^t \end{bmatrix}$$

Performing coordinate transformation from material axis to global axis, the constitutive equations for each constituent in the subcell can be obtained. Subsequently, expressing the in-plane stress components in terms of the in-plane strain and the out-of-plane stress components and expressing the out-of-plane strain components in terms of the in-plane stress and out-of-plane stress components, the effective stress-strain relations for the infinitesimal element (subcell) are obtained.

The incremental average stress-strain relations for a subcell can be written as follows:

$$\left\{\begin{array}{c} \mathrm{d}\varepsilon_i^{(\alpha,\beta)} \\ \hline \mathrm{d}\varepsilon_o^{(\alpha,\beta)} \end{array}\right\} = \left[\begin{array}{c|c} \bar{S}_i^t & \bar{S}_{io}^t \\ \hline \bar{S}_{oi}^t & \bar{S}_o^t \end{array}\right] \left\{\begin{array}{c} \mathrm{d}\sigma_i^{(\alpha,\beta)} \\ \hline \mathrm{d}\sigma_o^{(\alpha,\beta)} \end{array}\right\} \tag{6}$$

During the incremental-iterative solution scheme, a change in displacement takes place. The displacement increment causes an increment of strain $\Delta\{\bar{\varepsilon}\}$ (on the average basis for the micromaterial model). The material model is required to calculate the tangential stiffness matrix and the incremental average stress $\{\Delta\bar{\sigma}\}$. In this investigation the averaging procedure employed yields the tangential stiffness matrix and incremental stresses under an increment of strains (for more detail, see Ref. 13).

In-Plane Nonlinear Stress-Strain Relations

Material nonlinear behavior could be observed in in-plane and out-of-plane relations. However, it is much more important to consider the nonlinearity of the in-plane compliances in the in-plane relations than in the out-of-plane relations (nonlinearities in σ_{11}, σ_{22}, σ_{12} of the woven composite are much more pronounced than nonlinearities in σ_{13}, σ_{23}). The in-plane relations contribute significant stress resultants and therefore govern the loading carry capability of thin-walled composite structures. Any nonlinear stress-strain relations of the constituents can be applied in this methodology. However, the nonlinear elastic stress-strain relations used in this paper approximately represent most of the nonlinearity of polymer composites. The material nonlinearity, in fiber tows, considered in here is a modification of the one suggested by Hahn and Tsai.[14] Nonlinear shear strain-stress relations for fiber tows are assumed to be dominant. These relations are expressed in the following form:

$$(\gamma_{12})_k = (S_{66})_k(\sigma_{12})_k + (S_{6666})_k(\sigma_{12})_k^3, \qquad k = f, w \tag{7}$$

$$(\gamma_{13})_k = (S_{55})_k(\sigma_{13})_k + (S_{5555})_k(\sigma_{13})_k^3, \qquad k = f, w \tag{8}$$

For convenience and consistency with Hahn and Tsai's equation, the nonlinearity of the matrix material is simply assumed to have the same form as just shown. In the preceding equations S_{6666} and S_{5555} are the nonlinear stiffness terms, which are obtained experimentally.

In the process of deriving the nonlinear terms in the constitutive equations and the transformation from material to global axes, several assumptions have been made. To show these assumptions and how they affect the derived equations, consider the strain component ε_{xx} in the fill direction. Using a second-order tensor transformation, we have the following equation:

$$\varepsilon_x = c_f^2\varepsilon_1 + s_f^2\varepsilon_3 - s_f c_f \gamma_{13} \tag{9}$$

where $c_f = \cos\theta_f$, $s_f = \sin\theta_f$ and from the stress-strain relations, we have

$$\varepsilon_1 = S_{11}\sigma_1 + S_{12}\sigma_2 + S_{13}\sigma_3 \tag{10a}$$

$$\varepsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2 + S_{33}\sigma_3 \tag{10b}$$

where

$$\sigma_1 = c_f^2\sigma_x + s_f^2\sigma_z + 2c_f s_f \sigma_{xz} \tag{10c}$$

$$\sigma_2 = \sigma_y \tag{10d}$$

$$\sigma_3 = s_f^2\sigma_x + c_f^2\sigma_z - 2c_f s_f \sigma_{xz} \tag{10e}$$

$$\sigma_{13} = -c_f s_f \sigma_x + c_f s_f \sigma_z + \left(c_f^2 - s_f^2\right)\sigma_{xz} \tag{10f}$$

Inserting Eqs. (8) and (10) into Eq. (9), ε_x can be written as follows:

$$\varepsilon_x = (\underline{S}_{11})_f\sigma_x + (\underline{S}_{12})_f\sigma_y + (\underline{S}_{13})_f\sigma_z + (\underline{S}_{15})_f\sigma_{xz} - c_f s_f S_{5555}\left[-c_f s_f\sigma_x - c_f s_f \sigma_z + \left(c_f^2 - s_f^2\right)\sigma_{xz}\right]^3 \tag{11}$$

where $(\underline{S}_{mn})_f$ is a function of c_f, s_f, and $(S_{mn})_f$. In the preceding equation some of the terms can be neglected using the following assumptions:

1) The ultimate objective of the derived formulation is implementation in the finite element method, and therefore $\sigma_z = 0$ for shell finite elements.

2) Because of the balanced structure of the woven cell, $(\underline{S}_{15})_f$, $(\underline{S}_{25})_f$, $(\underline{S}_{35})_f$ are odd functions of θ_f and therefore drops out of the equation, for instance $(\underline{S}_{15})_f|_{-\theta_f} = -(\underline{S}_{15})_f|_{\theta_f}$ (see Fig. 2 for detailed schematics).

3) The contribution of the nonlinear higher-order σ_{xz} terms to ε_x is very small and therefore is neglected in subsequent expressions.

Then Eq. (11) becomes

$$\varepsilon_x = (\underline{S}_{11})_f\sigma_x + (\underline{S}_{12})_f\sigma_y + c_f^4 s_f^4 S_{5555}\sigma_x^3 \tag{12}$$

Therefore, the only nonlinear term retained is $c_f^4 s_f^4 S_{5555}\sigma_x^3$. The same operations and assumptions are used for deriving the expressions for ε_y, γ_{xy}, and for the warp and matrix constituents. Therefore, the in-plane stress-strain relations for off-axis fill tows, warp tows, and matrix can be written as follows:

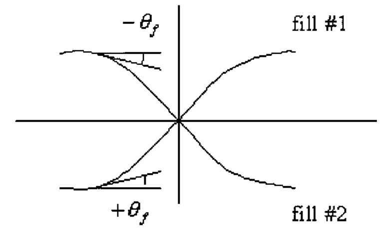

Fig. 2 Fill structure of a balanced representative cell.

Off-Axis Fill

$$
\begin{aligned}
(\varepsilon_x)_f &= (\underline{S}_{11})_f(\sigma_x)_f + (\underline{S}_{12})_f(\sigma_y)_f + (\underline{S}_{11n})_f(\sigma_x)_f^3 \\
(\varepsilon_y)_f &= (\underline{S}_{12})_f(\sigma_x)_f + (\underline{S}_{22})_f(\sigma_y)_f \\
(\varepsilon_{xy})_f &= (\underline{S}_{66})_f(\sigma_{xy})_f + (\underline{S}_{66n})_f(\sigma_{xy})_f^3
\end{aligned} \tag{13}
$$

where

$$(\underline{S}_{11})_f = c^4(S_{11})_f + 2c^2s^2(S_{13})_f + s^4(S_{33})_f + c^2s^2(S_{55})_f$$

$$(\underline{S}_{12})_f = c^2(S_{12})_f + s^2(S_{23})_f, \qquad (\underline{S}_{22})_f = (S_{22})_f$$

$$(\underline{S}_{66})_f = c^2(S_{66})_f + s^2(S_{44})_f, \qquad (\underline{S}_{11n})_f = c^4s^4(S_{5555})_f$$

$$(\underline{S}_{66n})_f = c^4(S_{6666})_f$$

where $c = \cos(\theta_f)$ and $s = \sin(\theta_f)$, where θ_f is the undulation angle, i.e., the angle between the fiber and the global x–y plane.

Off-Axis Warp

$$
\begin{aligned}
(\varepsilon_x)_w &= (\underline{S}_{11})_w(\sigma_x)_w + (\underline{S}_{12})_w(\sigma_y)_w \\
(\varepsilon_y)_w &= (\underline{S}_{12})_w(\sigma_x)_w + (\underline{S}_{22})_w(\sigma_y)_w + (\underline{S}_{22n})_w(\sigma_y)_w^3 \\
(\varepsilon_{xy})_w &= (\underline{S}_{66})_w(\sigma_{xy})_w + (\underline{S}_{66n})_w(\sigma_{xy})_w^3
\end{aligned} \tag{14}
$$

where

$$(\underline{S}_{22})_w = c^4(S_{11})_w + 2c^2s^2(S_{13})_w + s^4(S_{33})_w + c^2s^2(S_{55})_w$$

$$(\underline{S}_{12})_w = c^2(S_{12})_w + s^2(S_{23})_w, \qquad (\underline{S}_{11})_w = (S_{22})_w$$

$$(\underline{S}_{66})_w = c^2(S_{66})_w + s^2(S_{44})_w, \qquad (\underline{S}_{22n})_w = c^4s^4(S_{5555})_w$$

$$(\underline{S}_{66n})_w = c^4(S_{6666})_w$$

where $c = \cos(\theta_w)$ and $s = \sin(\theta_w)$, where θ_w has the same meaning as θ_f.

Matrix

$$
\begin{aligned}
(\varepsilon_{11})_m &= (S_{11})_m(\sigma_{11})_m + (S_{12})_m(\sigma_{22})_m \\
(\varepsilon_{22})_m &= (S_{12})_m(\sigma_{11})_m + (S_{22})_m(\sigma_{22})_m \\
(\gamma_{12})_m &= (S_{66})_m(\sigma_{12})_m + (S_{6666})_m(\sigma_{12})_m^3
\end{aligned} \tag{15}
$$

The incremental form of in-plane stress-strain relations of constituents may be written as

$$\mathrm{d}\begin{Bmatrix} (\sigma_x)_k \\ (\sigma_y)_k \\ (\sigma_{xy})_k \end{Bmatrix} = \begin{bmatrix} \left(C'_{11}\right)_k & \left(C'_{12}\right)_k & 0 \\ \left(C'_{12}\right)_k & \left(C'_{22}\right)_k & 0 \\ 0 & 0 & \left(C'_{66}\right)_k \end{bmatrix} \mathrm{d}\begin{Bmatrix} (\varepsilon_x)_k \\ (\varepsilon_y)_k \\ (\varepsilon_{xy})_k \end{Bmatrix} \tag{16}$$

Consider a quarter cell of the representative cell. Because of in-plane symmetry, this quarter cell represents the same mechanical properties as the whole cell. The quarter cell is further divided into many subcells, as shown in Fig. 1, for performing the averaging procedure. Once the incremental form of the stress-strain relations of the constituents are obtained from Eq. (16), the in-plane relations for a subcell can be derived. The equation is as follows:

$$\mathrm{d}\begin{Bmatrix}\sigma_x^{(\alpha,\beta)}\\ \sigma_y^{(\alpha,\beta)}\\ \sigma_{xy}^{(\alpha,\beta)}\end{Bmatrix} = \begin{bmatrix} C_{11}^{t(\alpha,\beta)} & C_{12}^{t(\alpha,\beta)} & 0\\ C_{12}^{t(\alpha,\beta)} & C_{22}^{t(\alpha,\beta)} & 0\\ 0 & 0 & C_{66}^{t(\alpha,\beta)}\end{bmatrix} \mathrm{d}\begin{Bmatrix}\varepsilon_x^{(\alpha,\beta)}\\ \varepsilon_y^{(\alpha,\beta)}\\ \varepsilon_{xy}^{(\alpha,\beta)}\end{Bmatrix} \tag{17}$$

In this procedure the average in-plane strains and stresses among subcells are assumed to have the following relationships (iso-stress; see Ref. 8 for explanations):

$$\mathrm{d}\sigma_x^{(\alpha,\beta)} = \mathrm{d}\sigma_x^{(\alpha^*,\beta)}$$

$$(\alpha = 1, \ldots, N_x - 1;\ \alpha^* = \alpha + 1;\ \beta = 1, \ldots, N_y) \tag{18a}$$

$$\mathrm{d}\sigma_y^{(\alpha,\beta)} = \mathrm{d}\sigma_y^{(\alpha,\beta^*)}$$

$$(\alpha = 1, \ldots, N_x;\ \beta = 1, \ldots, N_y - 1;\ \beta^* = \beta + 1) \tag{18b}$$

$$\mathrm{d}\sigma_{xy}^{(\alpha,\beta)} = \mathrm{d}\bar{\sigma}_{xy} \qquad (\alpha = 1, \ldots, N_x;\ \beta = 1, \ldots, N_y) \tag{18c}$$

$$\sum_{\alpha=1}^{N_x} \frac{l_x^{(\alpha,\beta)}}{L_x}\,\mathrm{d}\varepsilon_x^{(\alpha,\beta)} = \mathrm{d}\bar{\varepsilon}_x \qquad (\beta = 1, \ldots, N_y) \tag{18d}$$

$$\sum_{\beta=1}^{N_{yx}} \frac{l_y^{(\alpha,\beta)}}{L_y}\,\mathrm{d}\varepsilon_y^{(\alpha,\beta)} = \mathrm{d}\bar{\varepsilon}_y \qquad (\alpha = 1, \ldots, N_x) \tag{18e}$$

$$\sum_{\beta=1}^{N_y}\sum_{\alpha=1}^{N_x} \frac{l_x^{(\alpha,\beta)}}{L_x}\frac{l_y^{(\alpha,\beta)}}{L_y}\,\mathrm{d}\varepsilon_{xy}^{(\alpha,\beta)} = \mathrm{d}\bar{\varepsilon}_{xy} \qquad (\beta = 1, \ldots, N_y) \tag{18f}$$

The quantities with the bar denote the incremental average strain and stress components of the whole cell. N_x and N_y are the total number of subcells in x and y directions, respectively. The incremental average stresses induced by incremental average strains are expressed as

$$\mathrm{d}\{\bar{\sigma}\} = \sum_{\alpha=1}^{N_x}\sum_{\beta=1}^{N_y} \frac{l_x^{(\alpha,\beta)} l_y^{(\alpha,\beta)}}{L_x L_y}\,\mathrm{d}\{\sigma^{(\alpha,\beta)}\} \tag{19}$$

where L_x and L_y denote the length and width of the quarter cell, respectively. Equations (18) together with Eq. (17) provide sufficient information to distribute the incremental average strains to each subcell. Once the average strains in each subcell are known, the incremental average stresses of the cell can be obtained using Eqs. (17) and (19). Now by replacing the stress terms in Eqs. (18), and by using Eq. (17) and moving the terms in the right-hand side at the equality sign to the left, a simultaneous linear system of equations is formed. In this system the incremental strains of each subcell are unknown, and incremental average strains of the cell are known. The system is denoted by the following equation:

$$\mathrm{d}\{\varepsilon_i^{(\alpha,\beta)}\} = [A^{(\alpha,\beta)}]\,\mathrm{d}\{\bar{\varepsilon}_i\}$$

$$(\alpha = 1, \ldots, N_x \text{ and } \beta = 1, \ldots, N_y) \tag{20}$$

By combining Eqs. (17), (19), and (20), the incremental average stresses can be obtained:

$$\mathrm{d}\{\bar{\sigma}\} = \sum_{\alpha=1}^{N_x}\sum_{\beta=1}^{N_y} \frac{l_x^{(\alpha,\beta)} l_y^{(\alpha,\beta)}}{L_x L_y}\,[C^{(\alpha,\beta)}][A^{(\alpha,\beta)}]\,\mathrm{d}\{\bar{\varepsilon}\} \tag{21}$$

or

$$\mathrm{d}\{\bar{\sigma}\} = [\bar{C}]\,\mathrm{d}\{\bar{\varepsilon}\} \tag{22}$$

with

$$[\bar{C}] = \sum_{\alpha=1}^{N_x}\sum_{\beta=1}^{N_y} \frac{l_x^{(\alpha,\beta)} l_y^{(\alpha,\beta)}}{L_x L_y}\,[C^{(\alpha,\beta)}][A^{(\alpha,\beta)}] \tag{23}$$

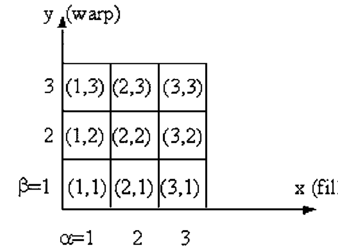

Fig. 3 Structure of a 3 × 3 quarter cell.

Equation (23) provides the tangential stiffness matrix for the average in-plane stress-strain relations. The incremental stresses calculated from Eq. (21) accumulate error when the average incremental strains are not small enough. To overcome this problem, one needs to update the strains in each subcell and then find the stresses of the constituents. The corresponding average stresses of the cell are directly determined by the following:

$$\{\bar{\sigma}\} = \sum_{\alpha=1}^{N_x}\sum_{\beta=1}^{N_y} \frac{l_x^{(\alpha,\beta)} l_y^{(\alpha,\beta)}}{L_x L_y}\{\sigma^{(\alpha,\beta)}\} \tag{24}$$

To demonstrate the preceding equations in detail, consider an example where the quarter cell is subdivided into 3 × 3 subcells as depicted in Fig. 3. One can insert Eq. (17) into Eq. (18a) to obtain the following:

$$\mathrm{d}\sigma_x^{(1,1)} = C_{11}^{(1,1)}\mathrm{d}\varepsilon_x^{(1,1)} + C_{12}^{(1,1)}\mathrm{d}\varepsilon_y^{(1,1)} = C_{11}^{(1,2)}\mathrm{d}\varepsilon_x^{(1,2)} + C_{12}^{(1,2)}\mathrm{d}\varepsilon_y^{(1,2)}$$
$$= C_{11}^{(1,3)}\mathrm{d}\varepsilon_x^{(1,3)} + C_{12}^{(1,3)}\mathrm{d}\varepsilon_y^{(1,3)} \tag{25a}$$

$$\mathrm{d}\sigma_x^{(2,1)} = C_{11}^{(2,1)}\mathrm{d}\varepsilon_x^{(2,1)} + C_{12}^{(2,1)}\mathrm{d}\varepsilon_y^{(2,1)} = C_{11}^{(2,2)}\mathrm{d}\varepsilon_x^{(2,2)} + C_{12}^{(2,2)}\mathrm{d}\varepsilon_y^{(2,2)}$$
$$= C_{11}^{(2,3)}\mathrm{d}\varepsilon_x^{(2,3)} + C_{12}^{(2,3)}\mathrm{d}\varepsilon_y^{(2,3)} \tag{25b}$$

$$\mathrm{d}\sigma_x^{(3,1)} = C_{11}^{(3,1)}\mathrm{d}\varepsilon_x^{(3,1)} + C_{12}^{(3,1)}\mathrm{d}\varepsilon_y^{(3,1)} = C_{11}^{(3,2)}\mathrm{d}\varepsilon_x^{(3,2)} + C_{12}^{(3,2)}\mathrm{d}\varepsilon_y^{(3,2)}$$
$$= C_{11}^{(3,3)}\mathrm{d}\varepsilon_x^{(3,3)} + C_{12}^{(3,3)}\mathrm{d}\varepsilon_y^{(3,3)} \tag{25c}$$

In the same way insert Eq. (17) into Eq. (18b) to obtain

$$\mathrm{d}\sigma_y^{(1,1)} = C_{12}^{(1,1)}\mathrm{d}\varepsilon_x^{(1,1)} + C_{22}^{(1,1)}\mathrm{d}\varepsilon_y^{(1,1)} = C_{12}^{(2,1)}\mathrm{d}\varepsilon_x^{(2,1)} + C_{12}^{(2,1)}\mathrm{d}\varepsilon_y^{(2,1)}$$
$$= C_{12}^{(3,1)}\mathrm{d}\varepsilon_x^{(3,1)} + C_{22}^{(3,1)}\mathrm{d}\varepsilon_y^{(3,1)} \tag{25d}$$

$$\mathrm{d}\sigma_y^{(1,2)} = C_{12}^{(1,2)}\mathrm{d}\varepsilon_x^{(1,2)} + C_{22}^{(1,2)}\mathrm{d}\varepsilon_y^{(1,2)} = C_{12}^{(2,2)}\mathrm{d}\varepsilon_x^{(2,2)} + C_{22}^{(2,2)}\mathrm{d}\varepsilon_y^{(2,2)}$$
$$= C_{12}^{(3,2)}\mathrm{d}\varepsilon_x^{(3,2)} + C_{22}^{(3,2)}\mathrm{d}\varepsilon_y^{(3,2)} \tag{25e}$$

$$\mathrm{d}\sigma_y^{(1,3)} = C_{12}^{(1,3)}\mathrm{d}\varepsilon_x^{(1,3)} + C_{22}^{(1,3)}\mathrm{d}\varepsilon_y^{(1,3)} = C_{12}^{(2,3)}\mathrm{d}\varepsilon_x^{(2,3)} + C_{12}^{(2,3)}\mathrm{d}\varepsilon_y^{(2,3)}$$
$$= C_{12}^{(3,3)}\mathrm{d}\varepsilon_x^{(3,3)} + C_{22}^{(3,3)}\mathrm{d}\varepsilon_y^{(3,3)} \tag{25f}$$

Then insert Eq. (17) into Eq. (18c) to obtain the following:

$$\mathrm{d}\sigma_{xy}^{(1,1)} = C_{66}^{(1,1)}\mathrm{d}\varepsilon_{xy}^{(1,1)} = C_{66}^{(1,2)}\mathrm{d}\varepsilon_{xy}^{(1,2)} = C_{66}^{(1,3)}\mathrm{d}\varepsilon_{xy}^{(1,3)} = \cdots$$
$$= C_{66}^{(3,3)}\mathrm{d}\varepsilon_{xy}^{(3,3)} \tag{25g}$$

Considering Eq. (25a), moving the terms in the right-hand side to the left, then we could get the following two equations:

$$C_{11}^{(1,1)}\mathrm{d}\varepsilon_x^{(1,1)} + C_{12}^{(1,1)}\mathrm{d}\varepsilon_y^{(1,1)} - C_{11}^{(1,2)}\mathrm{d}\varepsilon_x^{(1,2)} - C_{12}^{(1,2)}\mathrm{d}\varepsilon_y^{(1,2)} = 0 \tag{26a}$$

$$C_{11}^{(1,2)}\mathrm{d}\varepsilon_x^{(1,2)} + C_{12}^{(1,2)}\mathrm{d}\varepsilon_y^{(1,2)} - C_{11}^{(1,3)}\mathrm{d}\varepsilon_x^{(1,3)} - C_{12}^{(1,3)}\mathrm{d}\varepsilon_y^{(1,3)} = 0 \tag{26b}$$

The same operations are performed for Eqs. (25b–25f) to obtain the following 10 equations:

$$C_{11}^{(2,1)}\,\mathrm{d}\varepsilon_x^{(2,1)} + C_{12}^{(2,1)}\,\mathrm{d}\varepsilon_y^{(2,1)} - C_{11}^{(2,2)}\,\mathrm{d}\varepsilon_x^{(2,2)} - C_{12}^{(2,2)}\,\mathrm{d}\varepsilon_y^{(2,2)} = 0 \tag{26c}$$

$$C_{11}^{(2,2)}\,\mathrm{d}\varepsilon_x^{(2,2)} + C_{12}^{(2,2)}\,\mathrm{d}\varepsilon_y^{(2,2)} - C_{11}^{(2,3)}\,\mathrm{d}\varepsilon_x^{(2,3)} - C_{12}^{(2,3)}\,\mathrm{d}\varepsilon_y^{(2,3)} = 0 \tag{26d}$$

$$C_{11}^{(3,1)}\,\mathrm{d}\varepsilon_x^{(3,1)} + C_{12}^{(3,1)}\,\mathrm{d}\varepsilon_y^{(3,1)} - C_{11}^{(3,2)}\,\mathrm{d}\varepsilon_x^{(3,2)} - C_{12}^{(3,2)}\,\mathrm{d}\varepsilon_y^{(3,2)} = 0 \tag{26e}$$

$$C_{11}^{(3,2)}\,\mathrm{d}\varepsilon_x^{(3,2)} + C_{12}^{(3,2)}\,\mathrm{d}\varepsilon_y^{(3,2)} - C_{11}^{(3,3)}\,\mathrm{d}\varepsilon_x^{(3,3)} - C_{12}^{(3,3)}\,\mathrm{d}\varepsilon_y^{(3,3)} = 0 \tag{26f}$$

$$C_{11}^{(1,1)}\,\mathrm{d}\varepsilon_x^{(1,1)} + C_{12}^{(1,1)}\,\mathrm{d}\varepsilon_y^{(1,1)} - C_{12}^{(2,1)}\,\mathrm{d}\varepsilon_x^{(2,1)} - C_{22}^{(2,1)}\,\mathrm{d}\varepsilon_y^{(2,1)} = 0 \tag{26g}$$

$$C_{12}^{(2,1)}\,\mathrm{d}\varepsilon_x^{(2,1)} + C_{22}^{(2,1)}\,\mathrm{d}\varepsilon_y^{(2,1)} - C_{12}^{(3,1)}\,\mathrm{d}\varepsilon_x^{(3,1)} - C_{22}^{(3,1)}\,\mathrm{d}\varepsilon_y^{(3,1)} = 0 \tag{26h}$$

$$C_{12}^{(1,2)}\,\mathrm{d}\varepsilon_x^{(1,2)} + C_{22}^{(1,2)}\,\mathrm{d}\varepsilon_y^{(1,2)} - C_{12}^{(2,2)}\,\mathrm{d}\varepsilon_x^{(2,2)} - C_{22}^{(2,2)}\,\mathrm{d}\varepsilon_y^{(2,2)} = 0 \tag{26i}$$

$$C_{12}^{(2,2)}\,\mathrm{d}\varepsilon_x^{(2,2)} + C_{22}^{(2,2)}\,\mathrm{d}\varepsilon_y^{(2,2)} - C_{12}^{(3,2)}\,\mathrm{d}\varepsilon_x^{(3,2)} - C_{22}^{(3,2)}\,\mathrm{d}\varepsilon_y^{(3,2)} = 0 \tag{26j}$$

$$C_{12}^{(1,3)}\,\mathrm{d}\varepsilon_x^{(1,3)} + C_{22}^{(1,3)}\,\mathrm{d}\varepsilon_y^{(1,3)} - C_{12}^{(2,3)}\,\mathrm{d}\varepsilon_x^{(2,3)} - C_{22}^{(2,3)}\,\mathrm{d}\varepsilon_y^{(2,3)} = 0 \tag{26k}$$

$$C_{12}^{(2,3)}\,\mathrm{d}\varepsilon_x^{(2,3)} + C_{22}^{(2,3)}\,\mathrm{d}\varepsilon_y^{(2,3)} - C_{12}^{(3,3)}\,\mathrm{d}\varepsilon_x^{(3,3)} - C_{22}^{(3,3)}\,\mathrm{d}\varepsilon_y^{(3,3)} = 0 \tag{26l}$$

From Eq. (25g) the following eight equations for $\sigma_{xy}^{(\alpha,\beta)}$ can be obtained as follows:

$$C_{66}^{(1,1)}\,\mathrm{d}\varepsilon_{xy}^{(1,1)} - C_{66}^{(1,2)}\,\mathrm{d}\varepsilon_{xy}^{(1,2)} = 0 \tag{26m}$$

$$C_{66}^{(1,2)}\,\mathrm{d}\varepsilon_{xy}^{(1,2)} - C_{66}^{(1,3)}\,\mathrm{d}\varepsilon_{xy}^{(1,3)} = 0 \tag{26n}$$

$$C_{66}^{(2,1)}\,\mathrm{d}\varepsilon_{xy}^{(2,1)} - C_{66}^{(2,2)}\,\mathrm{d}\varepsilon_{xy}^{(2,2)} = 0 \tag{26o}$$

$$C_{66}^{(2,2)}\,\mathrm{d}\varepsilon_{xy}^{(2,2)} - C_{66}^{(2,3)}\,\mathrm{d}\varepsilon_{xy}^{(2,3)} = 0 \tag{26p}$$

$$C_{66}^{(3,1)}\,\mathrm{d}\varepsilon_{xy}^{(3,1)} - C_{66}^{(3,2)}\,\mathrm{d}\varepsilon_{xy}^{(3,2)} = 0 \tag{26q}$$

$$C_{66}^{(3,2)}\,\mathrm{d}\varepsilon_{xy}^{(3,2)} - C_{66}^{(3,3)}\,\mathrm{d}\varepsilon_{xy}^{(3,3)} = 0 \tag{26r}$$

$$C_{66}^{(1,3)}\,\mathrm{d}\varepsilon_{xy}^{(1,3)} - C_{66}^{(2,3)}\,\mathrm{d}\varepsilon_{xy}^{(2,3)} = 0 \tag{26s}$$

$$C_{66}^{(2,3)}\,\mathrm{d}\varepsilon_{xy}^{(2,3)} - C_{66}^{(3,3)}\,\mathrm{d}\varepsilon_{xy}^{(3,3)} = 0 \tag{26t}$$

Then from Eq. (18d) three equations for $\varepsilon_x^{(\alpha,\beta)}$ can be obtained as follows:

$$\frac{l_x^{(1,1)}}{L_x}\cdot\mathrm{d}\varepsilon_x^{(1,1)} + \frac{l_x^{(2,1)}}{L_x}\cdot\mathrm{d}\varepsilon_x^{(2,1)} + \frac{l_x^{(3,1)}}{L_x}\cdot\mathrm{d}\varepsilon_x^{(3,1)} = \mathrm{d}\bar{\varepsilon}_x \tag{26u}$$

$$\frac{l_x^{(1,2)}}{L_x}\cdot\mathrm{d}\varepsilon_x^{(1,2)} + \frac{l_x^{(2,2)}}{L_x}\cdot\mathrm{d}\varepsilon_x^{(2,2)} + \frac{l_x^{(3,2)}}{L_x}\cdot\mathrm{d}\varepsilon_x^{(3,2)} = \mathrm{d}\bar{\varepsilon}_x \tag{26v}$$

$$\frac{l_x^{(1,3)}}{L_x}\cdot\mathrm{d}\varepsilon_x^{(1,3)} + \frac{l_x^{(2,3)}}{L_x}\cdot\mathrm{d}\varepsilon_x^{(2,3)} + \frac{l_x^{(3,3)}}{L_x}\cdot\mathrm{d}\varepsilon_x^{(3,3)} = \mathrm{d}\bar{\varepsilon}_x \tag{26w}$$

Performing the same thing for Eq. (18e), three more equations for $\varepsilon_y^{(\alpha,\beta)}$ can be obtained as follows:

$$\frac{l_y^{(1,1)}}{L_y}\cdot\mathrm{d}\varepsilon_y^{(1,1)} + \frac{l_y^{(1,2)}}{L_y}\cdot\mathrm{d}\varepsilon_y^{(1,2)} + \frac{l_y^{(1,3)}}{L_y}\cdot\mathrm{d}\varepsilon_y^{(1,3)} = \mathrm{d}\bar{\varepsilon}_y \tag{26x}$$

$$\frac{l_y^{(2,1)}}{L_y}\cdot\mathrm{d}\varepsilon_y^{(2,1)} + \frac{l_y^{(2,2)}}{L_y}\cdot\mathrm{d}\varepsilon_y^{(2,2)} + \frac{l_y^{(2,3)}}{L_y}\cdot\mathrm{d}\varepsilon_y^{(2,3)} = \mathrm{d}\bar{\varepsilon}_y \tag{26y}$$

$$\frac{l_y^{(3,1)}}{L_y}\cdot\mathrm{d}\varepsilon_y^{(3,1)} + \frac{l_y^{(3,2)}}{L_y}\cdot\mathrm{d}\varepsilon_y^{(3,2)} + \frac{l_y^{(3,3)}}{L_y}\cdot\mathrm{d}\varepsilon_y^{(3,3)} = \mathrm{d}\bar{\varepsilon}_y \tag{26z}$$

Then considering Eq. (18f), one equation for $\varepsilon_{xy}^{(\alpha,\beta)}$ is obtained as follows:

$$\frac{l_y^{(1,1)}}{L_y}\cdot\frac{l_x^{(1,1)}}{L_x}\cdot\mathrm{d}\varepsilon_{xy}^{(1,1)} + \frac{l_y^{(1,2)}}{L_y}\cdot\frac{l_x^{(1,2)}}{L_x}\cdot\mathrm{d}\varepsilon_{xy}^{(1,2)} + \cdots + \frac{l_y^{(3,3)}}{L_y}\cdot\frac{l_x^{(3,3)}}{L_x}\cdot\mathrm{d}\varepsilon_{xy}^{(3,3)} = \mathrm{d}\bar{\varepsilon}_{xy} \tag{26aa}$$

By now there are 27 equations all together [Eqs. (26a–26aa)]. They form a linear system of equations with strains of each subcell as unknown and the average strains of the cell as known. These equations are presented in a matrix form by the following:

$$\begin{bmatrix} C_{11}^{(1,1)} & C_{12}^{(1,1)} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \frac{l_y^{(1,1)}}{L_y}\cdot\frac{l_x^{(1,1)}}{L_x} & \cdots & \frac{l_y^{(3,3)}}{L_y}\cdot\frac{l_x^{(3,3)}}{L_x} \end{bmatrix} \mathrm{d}\begin{Bmatrix} \varepsilon_x^{(1,1)} \\ \varepsilon_y^{(1,1)} \\ \varepsilon_{xy}^{(1,1)} \\ \vdots \\ \varepsilon_{xy}^{(3,3)} \end{Bmatrix}$$

$$= \mathrm{d}\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \bar{\varepsilon}_{xy} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \mathrm{d}\begin{Bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\varepsilon}_{xy} \end{Bmatrix} \tag{27}$$

Therefore, these equations can be written as follows:

$$[B]\cdot\mathrm{d}\{\varepsilon_i^{(\alpha,\beta)}\} = [K]\cdot\mathrm{d}\{\bar{\varepsilon}_i\} \tag{28}$$

where $[B]$ is a 27×27 matrix and $[K]$ is a 27×3 matrix. So, one can be solved for the incremental strains in each subcell as follows:

$$\mathrm{d}\{\varepsilon_i^{(\alpha,\beta)}\} = [B]^{-1}[K]\cdot\mathrm{d}\{\bar{\varepsilon}_i\} \tag{29}$$

Solution Algorithm

The steps necessary to calculate the constituent stresses of each subcell $\{\sigma^{(\alpha,\beta)}\}$ in Eq. (24) can be summarized by the following:

1) For the $n+1$st increment of global average strains $\mathrm{d}\{\bar{\varepsilon}\}^{n+1}$, the corresponding incremental strains of each subcell are determined by Eq. (20).

2) The total strains of each subcell are updated by the following equation:

$$\{\varepsilon^{(\alpha,\beta)}\}^{n+1} = \{\varepsilon^{(\alpha,\beta)}\}^{n} + \mathrm{d}\{\varepsilon^{(\alpha,\beta)}\}^{n} \tag{30}$$

3) The total stresses for each constituent corresponding to the total average strains of each subcell are determined by solving Eqs. (13–15). A nonlinear root-seeking scheme is applied to update the stresses of the constituents. In this formulation the Newton method is used. As an example, consider σ_{xy} of fill for demonstration of the implementation of the Newton method. We have

$$\gamma_{xy} = \underline{S}_{66}\sigma_{xy} + \underline{S}_{66n}\sigma_{xy}^3 \tag{31}$$

Let

$$f(\sigma_{xy}) = \gamma_{xy} - \underline{S}_{66}\sigma_{xy} - \underline{S}_{66n}\sigma_{xy}^3 \tag{32}$$

The iterative process of Newton's method for finding the roots of $f(\sigma_{xy})$ can be expressed as follows:

$$(\sigma_{xy})_{k+1} = (\sigma_{xy})_k - \frac{f[(\sigma_{xy})_k]}{f'[(\sigma_{xy})_k]} \tag{33}$$

The iterative solution is continued until the following condition is met:

$$|(\sigma_{xy})_{k+1} - (\sigma_{xy})_k| < \varepsilon$$

where ε is a very small value (0.0001). The same procedure is used for σ_x, σ_y, and the stresses for the warp and matrix constituents.

4) The total stresses of each subcell are then obtained from the following equation:

$$\left\{\sigma^{(\alpha,\beta)}\right\}^{n+1} = \bar{t}_m\left\{\sigma^{(\alpha,\beta)}\right\}_m^{n+1} + \bar{t}_f\left\{\sigma^{(\alpha,\beta)}\right\}_f^{n+1} + \bar{t}_w\left\{\sigma^{(\alpha,\beta)}\right\}_w^{n+1} \tag{34}$$

Other nonlinear material behavior of the constituents can also be considered in the presented averaging procedure; however, this paper deals only with nonlinearity in shear.

Results

As demonstration of the prediction of developed formulation, two examples are considered. The geometry and material properties for these examples are listed here, as obtained from Refs. 5 and 7. The material properties of constituents (GPa for moduli and GPa^{-3} for S_{6666}) for glass/polyamide are E_L, 41.2; E_T, 15.7; G_{LT}, 5.59; υ_{LT}, 0.30; ν_{TT}, 0.48; S_{6666}, 37.0 and for polyamide are E_m, 4.31; ν_m, 0.36; S_{6666}, 9.88.

Geometry parameters for glass/polyamide are the following: H, 0.224 mm; H_t, 0.224 mm; a, 0.4 mm; and b, 0.4 mm. The material properties of constituents (GPa for moduli and GPa^{-3} for S_{6666}) for graphite/epoxy are E_L, 132; E_T, 9.31; G_{LT}, 4.61; ν_{LT}, 0.28; ν_{TT}, 0.46; S_{6666}, 7.29 and for epoxy are E_m, 3.43; ν_m, 0.38; S_{6666}, 9.88.

The geometry parameters for graphite/epoxy are the following: H, 0.244 mm; H_t, 0.244 mm; a, 0.4 mm, and b, 0.4 mm. Here we assume that $(S_{6666})_f$ equals $(S_{5555})_f$. The first example considered is glass/polyamide woven composite. Figures 4 and 5 depict the in-plane shear nonlinearity for glass/polyamide and polyamide resin, respectively. The observation can be made that nonnegligible nonlinearity exists for these materials under shear loading. For a plain-weave composite made of these materials, the nonlinearity induced by the constituents can be obtained through the aforementioned averaging procedure. Figure 6 depicts the uniaxial stress vs strain relation for both linear and nonlinear material behavior. A slight nonlinearity for this material model is observed in the uniaxial direction. However, for in-plane shear, as shown in Fig. 7, severe nonlinearity between shear stress and strain exists. In these figures the linear behavior is obtained by setting $S_{6666} = 0$ in the derived equations. Figure 8 depicts the comparison of present formulation prediction with the prediction of Ref. 7 for the axial stress-strain relation. Excellent agreement is achieved. Reasonable convergence is obtained for the both cases considered with 5 × 5 subcells. The next example considered for verification is a graphite/epoxy woven composite. Figure 9 depicts the comparison of present formulation prediction with the prediction of Ref. 7 for the axial stress-strain

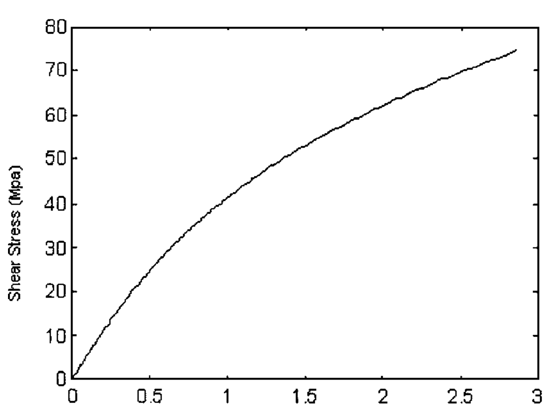

Fig. 4 In-plane shear stress-strain relation of glass/polyamide unidirectional laminaes.

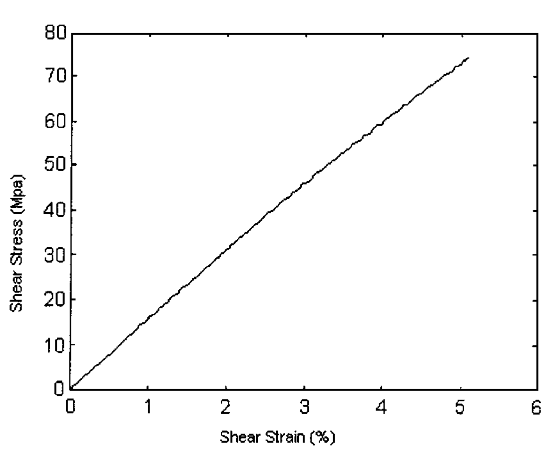

Fig. 5 In-plane shear stress-strain relation of polyamide matrices.

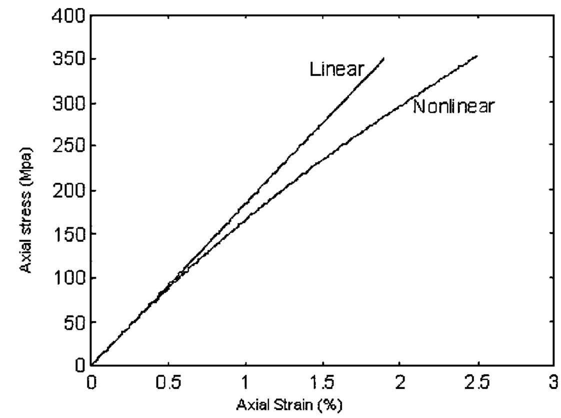

Fig. 6 In-plane nonlinear axial stress-strain relations of glass/polyamide fabric composites.

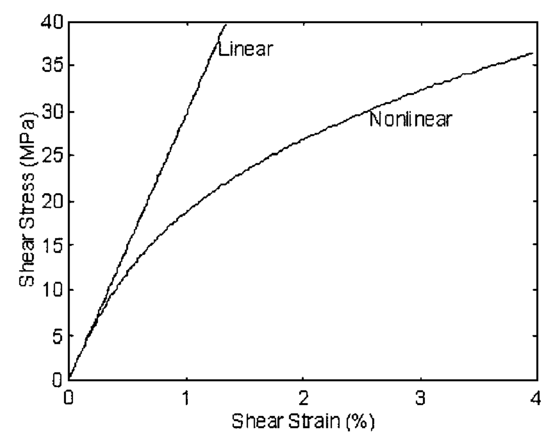

Fig. 7 In-plane nonlinear shear stress-strain relations of glass/polyamide fabric composites.

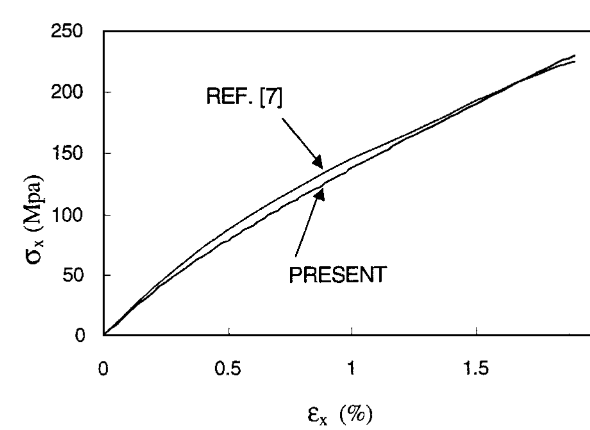

Fig. 8 Comparison of present prediction with Ref. 7 prediction for axial stress-strain relations for glass/polyimide woven composite.

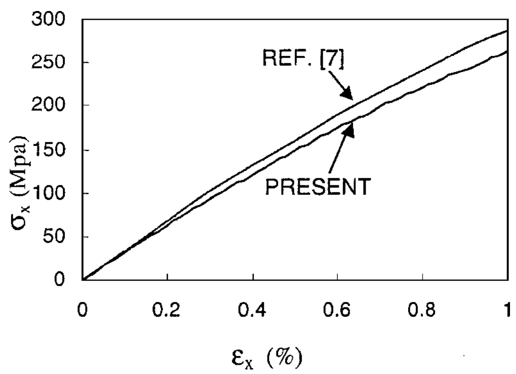

Fig. 9 Comparison of present prediction with Ref. 7 prediction for axial stress-strain relations for graphite/epoxy woven composite.

relation. In here, good agreement is also achieved. In this example only strain up to 1% is considered as the data reported in Ref. 7.

Conclusion

An averaging procedure is presented for homogenization of the stress-strain relations of plain-weave composites with material nonlinearity. A stress update procedure is developed. Linear and nonlinear stress vs strain behavior is obtained for both axial and shear directions. Some nonlinear behavior is observed in the axial direction. However, significant nonlinearity is observed in the shear direction. Material nonlinearity can have a significant effect on the global response of structures made of woven composites. The presented methodology can be directly applied to finite element softwares for structural analysis. The power and the usefulness of the developed formulation lies in the fact that stresses and strains can be found in each constituent of the subcell at each load increment. This will help in defining the effects of many parameters such as temperature, strain softening, strain rate, etc., on the mechanical properties of the woven composite and subsequently perform strength analysis and simulations.

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A. M. Waas
Associate Editor